Combinations (Part 1)

These notes are intended as a summary of section 8.4 (p. 721 – 726) in your workbook. You should also read the section for more complete explanations and additional examples.

How many 3 letter words can be formed from the word FACTOR?

Now consider 6 points on a plane, no three on the same line. Label them F, A, C, T, O, R. How many triangles can be drawn using these points as the vertices?

Notice that the points F, A, and C result in 6 permutations:

$\triangle FAC$	$\triangle ACF$	$\triangle CAF$
$\triangle FCA$	$\triangle AFC$	$\triangle CFA$

However, all 6 are the same triangle. The order of the vertices does not matter.

Since every triangle can be labelled in $3! = 6$ different ways, the number of different triangles should be found by dividing the number of permutations by $3!$.

$$\begin{aligned} \#\ of\ Triangles &= \frac{{}_6P_3}{3!} \\ &= \frac{120}{6} \\ &= 20 \end{aligned}$$

A **combination** is a selection of objects where the order does not matter. The number of combinations of n objects taken r at a time, written ${}_nC_r$ or $C(n,r)$, is given by the formula

$$
\begin{aligned}
{}_nC_r &= \frac{{}_n\mathrm{P}_r}{r!} \\
&= \frac{n!}{(n-r)!} \cdot \frac{1}{r!}
\end{aligned}
$$

$$
\boxed{{}_nC_r = \frac{n!}{r!(n-r)!}}
$$

Note: On your graphing calculator, press MATH then PRB then 3.

Example (not in workbook)
Evaluate algebraically, then verify using your calculator.

a) ${}_{15}C_1$

b) ${}_9C_9$

c) ${}_{12}C_7$

d) ${}_{12}C_5$

e) ${}_{18}C_{15}$

f) ${}_{84}C_4$

Example (not in workbook)
How many ways can a committee of 3 be chosen from 8 people?

Example 1 (sidebar p. 723)
In the Keno lottery, 20 numbers from 1 to 80 are chosen. How many combinations of 20 numbers are possible?

Example 2 (sidebar p. 724)
A local arena has 10 applicants interested in working in the snack bar.

a) How many ways can 4 applicants be chosen?

b) How many ways can 6 applicants be chosen?

Example 3 (sidebar p. 725)
A new store must hire 3 cashiers and 4 stock clerks. There are 7 applicants for cashier and 8 applicants for stock clerk. How many ways can the 7 employees be chosen?

Homework: #4, 5, 7, 8, 10, 12 – 14, 15a (algebraically), 15bcd (any method), 16 in the section 8.4 exercises (p. 727 – 732). Answers on p. 733.